Probabilistic AHP and TOPSIS for Multi-Attribute Decision-Making under Uncertainty

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Introduction

Making informed design decisions in the face of uncertainty is a common challenge in aerospace systems engineering.
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**Making informed design decisions in the face of uncertainty** is a common challenge in aerospace systems engineering.

### Uncertainty in Decision-Maker Preferences and System Performance

- **Credit:** Review of U.S. Human Spaceflight Plans Committee
- **Credit:** Timothy Simpson et al., Georgia Tech Systems Realization Lab

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### Trades among Multiple Objectives

- **Credit:** Dimitri Mavris, Georgia Tech Aerospace Systems Design Lab

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**Objective #1**

- **Better**
- **Worse**

**Objective Space**

- **Positive Ideal Design**
- **Negative Ideal Design**

**Objective #1**

- **Design #1** 1 1 7 3 1/7
- **Design #2** 1 1 3 2 3
- **Design #3** 1/7 1/3 1 1/9 1/2
- **Design #4** 1/3 1/2 9 1 5
- **Design #k** 7 1/3 2 1/5 1

**Objective #2**

- **Objective #3** 1/5 1/7 1 1/5 1/8
- **Objective #4** 1/2 1 5 1 1
- **Objective #n** 3 1 8 11

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**Objective #n**

- **Objective #1** 1 2 5 2 1/3
- **Objective #2** 1/2 1 7 1 1
- **Objective #3** 1/5 1/7 1 1/5 1/8
- **Objective #4** 1/2 1 5 1 1
- **Objective #n** 3 1 8 11

---

**Objective Space**

- **Positive Ideal Design**
- **Negative Ideal Design**
Common Tools for Multi-Attribute Decision-Making

- Analytic Hierarchy Process (AHP)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

AHP

- Developed by Saaty in mid-1970s
- Permits both prioritization of objectives and selection of designs based on a structured approach of pairwise comparisons
- Matrices are reciprocal and permit decision-maker(s) to evaluate consistency of preferences
- Using AHP for concept selection equivalent to simple additive weighting
- Conventional AHP is deterministic
Common Tools for Multi-Attribute Decision-Making

- Analytic Hierarchy Process (AHP)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS

- Developed by Yoon & Hwang, 1980
- Scores alternatives based on euclidean distances from positive and negative ideal designs

\[
C_i = \frac{S_i^-}{S_i^+ + S_i^-} = \frac{\| \vec{d}_i - \vec{d}_- \|}{\| \vec{d}_i - \vec{d}_+ \| + \| \vec{d}_i - \vec{d}_- \|}
\]

- Intuitive “physical” meaning
- Reflects diminishing marginal rates of substitution
- Does not provide method for weighting objectives (can be coupled with AHP)
Introduction

Objective
Facilitate informed decisions under uncertainty in decision-maker preferences by developing probabilistic adaptations of AHP and TOPSIS

Example Aerospace Application
Choose *launch vehicle* and circular *orbit* for a small (400 kg), responsive military reconnaissance satellite

- Targeted sensor with 1° FOV angle and 1.0 m GSD at 400 km reference altitude
- 110 kg/m² ballistic coefficient, minimal propellant available for orbit maintenance

Credits: USAF (top,right) NASA (left)
Methodology Summary

Problem Setup

1. Generate list of objectives/attributes
2. Generate list of candidate designs
3. Evaluate and record each design’s performance with respect to each attribute
4. Populate AHP objectives pairwise comparison matrix, including PDFs on each element

Monte Carlo Simulation

Repetitive Sub-Process

5. Select* an AHP objectives pairwise comparison matrix, based on PDFs for each element
6. Determine traditional deterministic AHP objectives priority vector (weightings)
7. Apply traditional deterministic TOPSIS
8. Record scores and rank order of alternatives

*Selections in this step are automated as part of the Monte Carlo simulation.

Results Visualization and Decision-Making

9. Visualize and Review Results
10. Select Alternative(s)
### Methodology: Problem Setup

**Step 1: Generate List of Objectives/Attributes**

What does the decision-maker wish to achieve?

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Units</th>
<th>Preferred Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Margin</td>
<td>percent</td>
<td>High</td>
</tr>
<tr>
<td>Launch Cost</td>
<td>$FY2009M</td>
<td>Low</td>
</tr>
<tr>
<td>Launch Reliability</td>
<td>percent</td>
<td>High</td>
</tr>
<tr>
<td>Image Field of View Area</td>
<td>km²</td>
<td>High</td>
</tr>
<tr>
<td>Image Nadir Ground Sample Distance</td>
<td>km</td>
<td>Low</td>
</tr>
<tr>
<td>Mean Worst-Case Daily Data Latency</td>
<td>hours</td>
<td>Low</td>
</tr>
<tr>
<td>Mean Daily Coverage Time</td>
<td>hours</td>
<td>High</td>
</tr>
<tr>
<td>Orbit Lifetime</td>
<td>years</td>
<td>High</td>
</tr>
</tbody>
</table>
Methodology: Problem Setup

Step 2: Generate List of Candidate Designs

*With what design choices can these objectives be influenced?*

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Options Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Vehicle</td>
<td>Falcon 1, Falcon 1e, Pegasus XL, Pegasus XL with HAPS, Taurus 2110, Taurus 2210,</td>
</tr>
<tr>
<td></td>
<td>Taurus 3110, Taurus 3210, Minotaur I, Minotaur IV, Athena I, Athena II</td>
</tr>
<tr>
<td>Orbit Altitude (km)</td>
<td>200, 300, 400, 600, 1000, 1500, 2000</td>
</tr>
<tr>
<td>Orbit Inclination (deg.)</td>
<td>0, 10, 20, 30, 40, 50, 60, 70, 80, 90</td>
</tr>
</tbody>
</table>

840 Discrete Design Options
Step 3: Evaluate each Design’s Performance with respect to each Attribute

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Units</th>
<th>Evaluation Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Margin</td>
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<tr>
<td>Launch Cost</td>
<td>$FY2009M</td>
<td>LV Database</td>
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<tr>
<td>Launch Reliability</td>
<td>Percent</td>
<td>LV Database</td>
</tr>
<tr>
<td>Image Field of View Area</td>
<td>km²</td>
<td>Geometry</td>
</tr>
<tr>
<td>Image Nadir Ground Sample Distance</td>
<td>km</td>
<td>Geometry</td>
</tr>
</tbody>
</table>

Filters:
- Launch Margin >0% and <100%
- Mean Daily Coverage >0 hrs.
- Orbit Lifetime >3 months

59 Candidate Designs
Step 4: Populate AHP Prioritization Matrix, including Uncertainties

**Baseline AHP Prioritization Matrix**

<table>
<thead>
<tr>
<th></th>
<th>High Launch Margin</th>
<th>Low Launch Cost</th>
<th>High Launch Reliability</th>
<th>High Image FOV Area</th>
<th>Low Image Nadir GSD</th>
<th>Low Mean Worst-Case Daily Data Latency</th>
<th>High Mean Daily Coverage Time</th>
<th>High Orbit Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Launch Margin</td>
<td>1</td>
<td>1/3</td>
<td>3</td>
<td>5</td>
<td>1/6</td>
<td>1/3</td>
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<td>3</td>
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<td>5</td>
<td>7</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2</td>
<td>5</td>
</tr>
<tr>
<td>High Launch Reliability</td>
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<td>1</td>
<td>3</td>
<td>1/8</td>
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<td>1</td>
<td>1/9</td>
<td>1/8</td>
<td>1/7</td>
<td>1/7</td>
</tr>
<tr>
<td>Low Image Nadir GSD</td>
<td>6</td>
<td>4</td>
<td>8</td>
<td>9</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>7</td>
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<td>3</td>
<td>5</td>
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<td>4</td>
<td>7</td>
<td>1/5</td>
<td>1/2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>High Orbit Lifetime</td>
<td>1/3</td>
<td>1/5</td>
<td>5</td>
<td>7</td>
<td>1/7</td>
<td>1/5</td>
<td>1/3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Verbal Description of Importance Ratings**

- 1 = Equal
- 3 = Weak
- 5 = Strong
- 7 = Very Strong
- 9 = Absolute

Methodology: Problem Setup

Step 4: Populate AHP Prioritization Matrix, including Uncertainties

| Actual AHP Scale, $a$ | 1/9 | 1/8 | 1/7 | 1/6 | 1/5 | 1/4 | 1/3 | 1/2 | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

Virtual AHP Scale, $v$: -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
Step 4: Populate AHP Prioritization Matrix, including Uncertainties

Virtual AHP Scale, $v$ | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

$\pm 2$
Methodology: Problem Setup

Step 4: Populate AHP Prioritization Matrix, including Uncertainties

\[ v(a) = \begin{cases} 
\frac{2a - 1}{a} & \text{if } a < 1 \\
2 - v & \text{if } v < 1 \\
a & \text{if } a \geq 1 \\
v & \text{if } v \geq 1 
\end{cases} \]

Uncertainty Matrix (Virtual Scale)

<table>
<thead>
<tr>
<th>Uncertainty Matrix (Virtual Scale)</th>
<th>High Launch Margin</th>
<th>Low Launch Cost</th>
<th>High Launch Reliability</th>
<th>High Image FOV Area</th>
<th>Low Image Nadir GSD</th>
<th>Low Mean Worst-Case Daily Data Latency</th>
<th>High Mean Daily Coverage Time</th>
<th>High Orbit Lifetime</th>
</tr>
</thead>
<tbody>
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<td>3.5</td>
<td>4.0</td>
<td>5.0</td>
<td>5.5</td>
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<tr>
<td>Low Launch Cost</td>
<td>1.5</td>
<td>1.5</td>
<td>2.0</td>
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<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
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<tr>
<td>High Launch Reliability</td>
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<td>2.0</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>High Image FOV Area</td>
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<td>3.0</td>
<td>1.5</td>
<td>1.5</td>
<td>0.5</td>
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<tr>
<td>Low Image Nadir GSD</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Low Mean Worst-Case Daily Data Latency</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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<td>0.5</td>
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<tr>
<td>High Mean Daily Coverage Time</td>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>High Orbit Lifetime</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</tbody>
</table>
Methodology: Problem Setup

Step 4: Populate AHP Prioritization Matrix, including Uncertainties

**Probabilistic Prioritization Matrix (Virtual Scale)**

<table>
<thead>
<tr>
<th></th>
<th>High Launch Margin</th>
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<th>High Mean Daily Coverage Time</th>
<th>High Orbit Lifetime</th>
</tr>
</thead>
<tbody>
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<td>2-X₁,₂</td>
<td>X₁,₃</td>
<td>X₁,₄</td>
<td>X₁,₅</td>
<td>X₁,₆</td>
<td>X₁,₇</td>
<td></td>
</tr>
<tr>
<td>Low Launch Cost</td>
<td></td>
<td>1</td>
<td>X₂,₃</td>
<td>X₂,₄</td>
<td>X₂,₅</td>
<td>X₂,₆</td>
<td>X₂,₇</td>
<td></td>
</tr>
<tr>
<td>High Launch Reliability</td>
<td></td>
<td>2-X₁,₃</td>
<td>1</td>
<td>X₃,₄</td>
<td>X₃,₅</td>
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</tr>
<tr>
<td>High Image FOV Area</td>
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<td>2-X₁,₄</td>
<td>2-X₂,₄</td>
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<td>X₄,₅</td>
<td>X₄,₆</td>
<td>X₄,₇</td>
<td></td>
</tr>
<tr>
<td>Low Image Nadir GSD</td>
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<td>2-X₁,₅</td>
<td>2-X₂,₅</td>
<td>2-X₃,₅</td>
<td>2-X₄,₅</td>
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<td>Low Mean Worst-Case Daily Data Latency</td>
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<td>2-X₁,₆</td>
<td>2-X₂,₆</td>
<td>2-X₃,₆</td>
<td>2-X₄,₆</td>
<td>2-X₅,₆</td>
<td>1</td>
<td>X₆,₇</td>
</tr>
<tr>
<td>High Mean Daily Coverage Time</td>
<td></td>
<td>2-X₁,₇</td>
<td>2-X₂,₇</td>
<td>2-X₃,₇</td>
<td>2-X₄,₇</td>
<td>2-X₅,₇</td>
<td>2-X₆,₇</td>
<td>1</td>
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<tr>
<td>High Orbit Lifetime</td>
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<td>2-X₁,₈</td>
<td>2-X₂,₈</td>
<td>2-X₃,₈</td>
<td>2-X₄,₈</td>
<td>2-X₅,₈</td>
<td>2-X₆,₈</td>
<td>2-X₇</td>
</tr>
</tbody>
</table>

\[ X_{ji} = 2 - X_{ij} \]
Methodology: Monte Carlo Sim

For each Simulation:

**Step 5:** Select an AHP Prioritization Matrix

**Step 6:** Determine AHP Priority Vector

**Step 7:** Compute TOPSIS Score

**Step 8:** Record Scores, Ranks of Alternatives
Methodology: Decision-Making

Step 9: Visualize and Review Results

TOPSIS Score

- Falcon 1e – 400 km, 70°
- Falcon 1e – 400 km, 80°
- Falcon 1e – 400 km, 90°
- Taurus 2210 – 400 km, 90°
Methodology: Decision-Making

Step 9: Visualize and Review Results

Among the 10,000 Monte Carlo runs, how often does a particular design appear as the best alternative?

- Falcon 1e - 400 km, 90°: 88.71% probability of being the Top Design, Deterministic TOPSIS Score = 0.6194
- Falcon 1e - 400 km, 70°: 5.82% probability, Deterministic TOPSIS Score = 0.6166
- Falcon 1e - 400 km, 80°: 5.42% probability, Deterministic TOPSIS Score = 0.6183
- Taurus 2210 - 400 km, 90°: 0.05% probability, Deterministic TOPSIS Score = 0.6134
Methodology: Decision-Making

Step 9: Visualize and Review Results

Among the 10,000 Monte Carlo runs, how often does a particular design appear among the top N alternatives?
**Methodology: Decision-Making**

**Step 9: Visualize and Review Results**

Among the 10,000 Monte Carlo runs, how often does a particular design appear among the top N alternatives?

---

**“Probability Reversal”**

- **Design 6**: Minotaur I - 400 km, 50°
- **Design 8**: Minotaur I - 400 km, 60°
- **Design 10**: Falcon 1e - 400 km, 70°
- **Design 11**: Minotaur I - 400 km, 70°
- **Design 12**: Athena I - 400 km, 70°
- **Design 13**: Falcon 1e - 400 km, 80°
- **Design 14**: Minotaur I - 400 km, 80°
- **Design 15**: Athena I - 400 km, 80°
- **Design 16**: Falcon 1e - 400 km, 90°
- **Design 17**: Taurus 2210 - 400 km, 90°
- **Design 18**: Minotaur I - 400 km, 90°
- **Design 19**: Athena I - 400 km, 90°
- **Design 30**: Falcon 1e - 600 km, 70°
- **Design 34**: Falcon 1e - 600 km, 80°
- **Design 37**: Falcon 1e - 600 km, 90°
Methodology: Decision-Making

Step 10: Select Alternative(s)

Considerations

- Did particular design characteristics appear consistently in top designs?
- Is finer discretization on continuous variables necessary, and is there time for a second iteration?
- Is selection intended to result in a single design solution or a family of solutions?
Conclusion

- Probabilistic extension of AHP and TOPSIS can facilitate more informed engineering decisions under uncertainty
  - Adds easily interpretable results to characterize effects of uncertainty in preferences
  - May be particularly useful if multiple designs must be selected
  - Demonstrated on practical engineering application

- Comprehensive method consists of 10 steps and 3 segments:
  - **Problem Setup**: Definition of objectives, priorities, uncertainties, design attributes, and candidate designs; and evaluation of each design with respect to attributes
  - **Monte Carlo Simulation**: Application of traditional AHP and TOPSIS to thousands of “dispersed” AHP prioritization matrices
  - **Results Visualization and Decision-Making**: Examination of results through up to four techniques to inform final decision

- Areas for Expansion
  - Incorporation of design attribute uncertainty (model uncertainty)
  - Incorporation of nonsymmetric or non-triangular PDFs
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